

POLYNOMIALSIntroduction

1. "Term" : A Mathematical symbol consisting of a sign part, Numerical (Number) Part and a variable part is called a "Term"

Example : $2x$, -3 , $\frac{1}{2}x^2$, x^3 .etc

TERM	NAME	Sign Part	Numerical Part	Variable Part	Degree	Coefficient
2	Constant Term		2	x^0	0	+2
$-2x$	x- term		2	x	1	-2
$-5/6 x^2$	x^2 – term		5/6	x^2	2	-5/6
$1/3 x^3$	x^3 - term		1/3	x^3	3	+1/3

Note

- * Terms are named after (based on) their variable parts.
- * Every term has a co-efficient, base and degree (index, power, exponent, radical).
- * The Co-efficient is the combination of sign part and the numerical part of a term.
- * **Sentence:** A group of terms is called a sentence.
Ex: $x+2$, x^2+7x+2
- * **Open sentence:** Any sentence which consists of at least one variable in it is called an open sentence.

Ex: $x+1=0$, x^2+2x+3

Note:

- * **Expressions and Equations are the two types of open sentences.**
- * **Expression :** An open sentence which doesn't consist of equality sign ($=$) is called an expression.
Ex: $x+2$, x^2-3 , x^3-x^2+3x+1 .
- * **Equation :** An open sentence which consists of equality sign ($=$) is called an equation.
Ex: $x+2=0$, $x^2-3=0$
- * **Polynomial :** Any nth degree expression is called a polynomial

Note:

- (i) Any polynomial which is written in the descending orders of x is called

the standard form or the general form. of a polynomial and it is denoted by $f(x)$, $p(x)$, $g(x)$, $h(x)$ etc.

i.e $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^2 + \dots + a_0 x^0$ where

$a_n, a_{n-1}, a_{n-2}, \dots, a_0$ are called the real coefficients of $x^n, x^{n-1}, \dots$ respectively.

Types of Polynomials

Based on highest degree

- * **Linear Polynomial:** Any polynomial which is in the form of $ax + b$, where $a \neq 0$ and a, b being real numbers is called a linear polynomial.

Ex : $2x + 3, 5x + 4, -5x - 3$

Note :

- * ' $ax + b$ ' is also called the standard form or general form of a linear polynomial.
- * Every linear polynomial has two terms.
- * The first term ' ax ' is called the x -term.
- * The second term b is called the x^0 term (or) constant term (or) independent term.
- * The highest degree of every linear polynomial is '1' (one).
- * Every linear polynomial has only one solution called factor (itself).
- * **Quadratic Polynomial** : Any polynomial which is in the form of $ax^2 + bx + c$ where $a \neq 0$ and a, b, c being real numbers is called a quadratic polynomial.

Example: $x^2 + 3x + 2, 2x^2 - 5x - 7, x^2 - 5x + 6$ etc.

Note :

- * $ax^2 + bx + c$ is called the standard form (general form) of a quadratic polynomial.
- * Every quadratic polynomial has three terms in it.
- * The first term ax^2 is called ' x^2 ' term.
- * The second term bx is called ' x ' term.
- * The third term c is called x^0 term or constant term or independent term.
- * Every quadratic polynomial has exactly two solutions called factors
- * If the two zeros of a quadratic polynomial $f(x)$ are denoted by α, β , then the formula to find it is given by
 $f(x) = k [x^2 - (\text{sum of the zeros}) x + (\text{product of the zeros})]$; i.e.

$f(x) = k [x^2 - (\alpha + \beta)x + \alpha\beta]$; where the relationship between the zeros and the co-efficients is given by

$$\alpha + \beta = -\frac{\text{co-efficient of } x}{\text{co-efficient of } x^2} = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{\text{constant term}}{\text{co-efficient of } x^2} = +\frac{c}{a}$$

- * **Cubic polynomial**: Any polynomial which is in the form of $ax^3 + bx^2 + cx + d$, where $a \neq 0$ and a, b, c, d being real numbers is called a cubic polynomial.

Ex: $x^3 - 3x^2 - 3x + 1, x^3 + 5x^2 - 2x + 10$

Note:

- * $ax^3 + bx^2 + cx + d$ is also called the standard form of a cubic polynomial.
 - * Every cubic polynomial consists of 4 terms.
 - * The first term ax^3 is called x^3 term.
 - * The second term bx^2 is called x^2 term.
 - * The third term cx is called x term.
 - * The fourth term d is called x^0 term or constant term or independent term.
 - * Every cubic polynomial has exactly 3 solutions called factors.
 - * If the three zeros of a cubic polynomial $f(x)$ are denoted by α, β, γ , then the formula to find it is given by
- $f(x) = k [x^3 - (\text{sum of the zeros})x^2 + (\text{sum of product of zeros taken two at a time})x - (\text{product of the zeros})]$; i.e.
- $f(x) = k [x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$; where the relationship between the zeros and the co-efficients is given by:

$$\alpha + \beta + \gamma = -\frac{\text{co-efficient of } x^2}{\text{co-efficient of } x^3} = -\frac{b}{a} \quad \text{and} \quad \alpha\beta\gamma = -\frac{\text{constant term}}{\text{co-efficient of } x^3} = -\frac{d}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = -\frac{\text{co-efficient of } x}{\text{co-efficient of } x^3} = \frac{c}{a}$$

- * **Biquadratic Polynomial**: Any polynomial which is in the form of $ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d, e , being Real numbers, $a \neq 0$ is called a biquadratic polynomial.

Ex: $6x^4 + 3x^3 - 2x^2 + 6x + 1$

Note:

- The standard form of a biquadratic polynomial is $ax^4 + bx^3 + cx^2 + dx + e$.
- Every biquadratic polynomial has 5 terms.

- The first term ax^4 is called ' x^4 ' term.
- The second term bx^3 is called ' x^3 ' term.
- The third term cx^2 is called ' x^2 ' term.
- The fourth term dx is called ' x ' term.
- The fifth term e is called ' x^0 ' term or constant or independent term.
- Every biquadratic polynomial has exactly 4 solutions called factors.

Types of Polynomials

Based on number of terms

- **Monomial**: Any polynomial which has 1 term is called a monomial.
Ex: x^2 , $2xy$, $-x$, xyzetc.
- **Bionomial**: Any polynomial which has 2 terms is called a Bionomial
Ex: $x + 2$, $3x^2 + 5$, $4x^2 + 5$, $4x^3 + 2x^2$ etc.
- **Trinomial**: Any polynomial which has 3 terms is called a trinomial
Ex: $x^2 + 3x + 2$, $x + y + z$, etc.
- **Zero of a polynomial**: If $f(x)$ be any given polynomial and ' a ' being a real number and if $f(a)=0$ then a is called the zero of the polynomial $f(x)$.
- **Factorisation**: A procedure (method) in which a given polynomial is expressed as the product of its maximum number of factors is known as factorisation.
- **Solve** : Finding all the possible values of the unknown quantity or quantities, which exists in the problem is called solving.
- **Remainder Theorem**: If $f(x)$ be any given polynomial of degree greater than or equal to 1 and small ' a ' be any real number, such that $f(x)$ is divided by $(x - a)$, then the remainder is equal to $f(x)$.

$$f(x) = (x - a) q(x) + r(x)$$

Note:

- * $f(x)$ is called the dividend
- * $(x - a)$ is called the divisor
- * $q(x)$ is called the quotient
- * $r(x)$ is called the remainder.

- * If a polynomial $p(x)$ is divided by $(x + a)$ the remainder is the value of $p(x)$ at $x = -a$ i.e., $p(-a)$.
 $[x + a = 0 \Rightarrow x = -a]$
- * If a polynomial $p(x)$ is divided by $ax - b$ the remainder is the value of $p(x)$ at $x = -a$ i.e., $p(-a)$.
 $[x + a = 0 \Rightarrow x = -a]$
- * If a polynomial $p(x)$ is divided by $ax + b$ the remainder is the value of $p(x)$ at $x = -b/a$ i.e., $p(-b/a)$.
- * If a polynomial $p(x)$ is divided by $(b - ax)$ the remainder is the value of $p(x)$ at $x = b/a$ i.e., $p(b/a)$.
 $[b - ax = 0 \Rightarrow -ax = -b \Rightarrow x = -b/-a \Rightarrow x = b/a]$
- * Remainder theorem is used to find the remainder.
- * Remainder theorem is also called the division rule i.e.
Dividend = Division $\times$ Quotient + Remainder.
- * **Factor Theorem** : If $f(x)$ be any given polynomial of degree greater than or equal to 1 and small 'a' be any real number, such that $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

Note:

- * $(x + a)$ is a factor of a polynomial $f(x)$ if $f(-a) = 0$
- * $(ax - b)$ is a factor of a polynomial $f(x)$ if $f(b/a) = 0$
- * $(ax + b)$ is a factor of a polynomial $f(x)$ if $f(-b/a) = 0$.
- * $(x - a)$ is a factor of a polynomial $f(x)$ if $f(a) = 0$
- * Factor theorem is used to factorize a polynomial by division method or Horner method.
